

# Adjusting Logit Model Estimation Results Obtained with Stated Preference Data

Mina Hassanvand PhD

**Abstract**— Stated preference (SP) surveys are commonplace in transportation engineering and planning. When the data from such surveys are used to estimate logit choice models, several refinements can be applied to account for limitations or departures from the assumptions made in development of the logit model and the estimation techniques used with it. This paper has reviewed three relevant refinements and has applied them in a case study using data from an SP survey where three of these adjustments are considered and the results are compared with the unadjusted results. Literature review showed there were not many studies with clear explanations regarding the nature and use of all three adjustments in common notation all together for SP data. Most papers focus on panel data and not SP data issues. This work can be beneficial to researchers and students as the three adjustments have been explained in detail using common notation in one single case study. The first adjustment accounts for the effects of using a sample with differential sampling rates across the population. The second one accounts for using a sample with differences in the probability of being included in the choice set. The third accounts for the effects of a sample with more than one observation from individuals. The SP data herein concern choice among alternative passenger modes for a trip given that car drive alone is not available, including different arrangements where friends or family members provide chauffeuring assistance at various levels of inconvenience. The intention of this work is to review and explain all concepts and compare the impacts of the three adjustments when done separately versus all together in a single case study involving SP data. This is important as with applying all three adjustments together as opposed to applying them separately or ignoring one, the fit of the model can improve and there could be a statistically significant change in the values of the t-statistics and t-ratios of the parameters and constants.

**Index Terms**— Logit Model Estimation; Stated Preference; Multiple Observations Per Respondent; Sample Observation Weights; Probability of Appearance in Choice Sets; Jackknife Estimation Technique.

## 1 INTRODUCTION

Stated preference (SP) analysis enjoys widespread use in support of transportation modeling and analysis. The basic approach is to conduct a survey of choice behavior concerning a set of hypothetical alternatives, use the resulting data to estimate logit models of this choice behavior, and then apply the estimated models or interpret the indications regarding sensitivities provided by the statistical estimates of the model parameter. The flexibility of the SP approach allows for complex data structures and survey techniques to be used. But the resulting data may be such that, in the strictest sense, more is required than simply running the standard logit estimation process available “right out of the box.”

Many researchers ignore the corrections, or ‘refinements’, and assume that the simple basic (or even “naïve”) analysis is sufficient

[1]. Even though accounting for these factors have gradually become more prevalent, there still exists a need for consistency in relevant notation and a better understanding of some of the technical issues and challenges in estimating logit models.

This paper presents a case study where the data from an SP survey of mode choice are used to perform logit model estimations where the model specification is held constant while three potential refinements are applied individually and in combination. These refinements include applying weights to the observations to account for differences across observations in the sampling rates, adjusting alternative specific constants to account for differences across alternatives in the probability of being included in the set of available alternatives which is apparent in the SP surveys, and applying a jackknife estimation technique to account for the correlations in the error structure expected with multiple observations from each respondent which is again apparent in the SP surveys.

Each of these refinements is covered in the existing literature, with the issue and the correction set out in each case. But they are each considered somewhat in isolation, using differing notation, and the material does not

---

Author Mina Hassanvand has obtained her Master's and PhD degrees in transportation engineering from the University of Calgary, Alberta in Canada and can be reached at e-mail: ghassanv@ucalgary.ca.

seem to be readily accessible to many of those using SP data. On this basis, it is felt that a paper is warranted presenting all three techniques in a consistent manner, covering theory and process, on a specific case study. The intention is to help disseminate greater understanding and capabilities in the use of the SP approach and the analysis of choice behavior using logit models more generally by showing improvements in model fit, t-statistics, and t-ratio values.

## 2 STUDY FRAMEWORK

1. Review each adjustment separately and include past work covering the relevant topics;
2. Describe the case study SP survey and the resulting data;
3. Describe basic tree structure chosen for the data to be analyzed using ALOGIT estimation software;
4. Present logit estimation results with different combinations of the refinements applied including:
5. No adjustments;
6. Adjustment to account for differential sampling rates across the population;
7. Adjustment to account for using a sample with differences in the probability of being included in the choice set;
8. Adjustment to account for the effects of a sample with more than one observation from individuals;
9. All adjustments applied together; and
10. Offer conclusions arising from the work.

## 3 LOGISTIC MODELS

Here a review of the literature on the three refinements along with presentation of selected material on each is done. This is achieved using a consistent set of terms and variable definitions which can help the readers as the notations all differ in past work. A brief review of the standard logit model and the basic estimation process is also provided as a 'point of entry' for some readers and in order to set out some of the consistent set of terms and variable definitions. Most of the literature have not used one or two of the adjustments. For example, in a work by [2] only the adjustment for differential sampling rates across the population and adjustment for multiple observations per individual has been considered. Other works by [3], [4], [5], and [6]

have used the adjustment for multiple observations per individual or the adjustment to account for differential sampling rates across the population. There are some research papers that have discussed adjustments discussed herein – in conjunction with revealed preference RP data – such as the work done by [7] and the work of [8]. It is notable to mention that correcting for sample weights is not necessarily tied to SP surveys and is implemented in commercial software packages such as ALOGIT and STATA. In some cases, especially for SP surveys, the target sample is obtained from a convenient sample (i.e. online panel) and quota sampling methods are used to match the population distribution for certain variables. This process minimizes the need for sample rate adjustments and has not been mentioned in some research papers.

In practice, correcting for the frequency of alternatives is usually addressed by setting availability conditions of alternatives for each SP scenario. The estimation of the joint likelihood is necessary since the observations in that case are no longer independent while realizing that this adjustment affects the standard errors of the estimated coefficients. In addition, the scale parameter can be parametrized with individuals' attributes to capture heteroskedasticity in their responses. In a work by [9] a fuzzy logic approach to such refinements has been considered. Also, in a work by [10] a quantum approach has been discussed. However, the consolidated approach found in [11] is the most widely used in practice. Since the study herein is concerned with travel in the City of Calgary, in Alberta Canada, an example of an aggregate travel mode share chart is provided in Figure 1.

### 3.1 Standard Logit Model and Basic Estimation Process

In the well-known standard logit model, each individual,  $i$ , in a population,  $I$ , choosing one alternative  $a$  out of a set of alternatives  $a \in A_i$ , associates a utility value  $U_{ai}$  with each alternative and selects the alternative with the highest utility value.

The utility value for each alternative is assumed to be influenced by the attributes of the alternative and the characteristics of the individual, with a linear-in-parameters

formulation and a randomly varying error component, as seen in Formula 1 and 2 found in work by [11]:

$$U(a, i) = \beta_{0ai} + \sum_{k \in K_a} \beta_{kai} x_{kai} + \varepsilon_{ai} \quad (1)$$

$$= V_{ai} + \varepsilon_{ai} \quad (2)$$

where:

$V_{ai}$  = measurable conditioning component of utility individual  $i$  associates with alternative  $a$   
 $\beta_{kai}$  = utility function parameter, or 'sensitivity', associated with measured attribute  $k$  for alternative  $a$  for individual  $i$   
 $\beta_{0ai}$  = utility function constant parameter, or 'constant', associated with alternative  $a$  for individual  $i$   
 $k$  = index for measured attributes  
 $K_a$  = set of measured attributes for alternative  $a$   
 $\varepsilon_{ai}$  = randomly varying error component of utility of individual  $i$  associated with alternative  $a$

With the assumption that the  $\varepsilon_{ai}$  vary according to identical and independent standard Gumbel distributions across all  $a \in A_i$  where the Standard Gumbel distribution has the cumulative distribution function as seen in Formula 3 by [12]:

$$F(x) = P(X \leq x) = e^{-e^{\lambda(x-\eta)}} \quad (3)$$

where:

$\eta$  = the location parameter of the distribution, which is also the mode of the distribution

$\lambda$  = the scale parameter of the distribution

Then the probability that a given alternative  $a^*$  has the greatest utility and therefore is the one chosen by the individual  $i$  is as seen in Formula 4:

$$P(a_i^*) = \frac{\exp[\lambda V_{a_i^*}]}{\sum_{a \in A_i} \exp[\lambda V_{a_i}]} \quad (4)$$

where:

$V_{ai}$  = utility of a choice alternative  
 $\lambda$  = the scale parameter of the distribution

This is indeed all well-known. Given a dataset of observations of selections of  $a_i^*$  from the set of alternatives  $A_i$  for a set of individuals  $i \in I$ , a set of estimates for the utility function parameters factored by the dispersion parameter, including the constant  $\lambda \beta_{0a}$  and the vector of sensitivities  $\lambda \beta_{ka} \forall k \in K_a$ , is established for each alternative for the set of individuals  $I$  using the likelihood function as seen in Formula 5:

$$L = \prod_{i \in I} P(a_i^*) = \prod_{i \in I} \left( \frac{\exp[\lambda V_{a_i^*}]}{\sum_{a \in A_i} \exp[\lambda V_{a_i}]} \right) = \prod_{i \in I} \left( \frac{\exp[\lambda \beta_{0a^*} + \sum_{k \in K_{a^*}} \lambda \beta_{ka^*} x_{ka_i^*}]}{\sum_{a \in A_i} \exp[\lambda \beta_{0a} + \sum_{k \in K_a} \lambda \beta_{ka} x_{ka}]} \right) \quad (5)$$

and for computational tractability this is converted into log formulation as seen in Formula 6:

$$LL = \ln(L) = \sum_{i \in I} \ln(P(a_i^*)) = \sum_{i \in I} \ln \left( \frac{\exp[\lambda \beta_{0a^*} + \sum_{k \in K_{a^*}} \lambda \beta_{ka^*} x_{ka_i^*}]}{\sum_{a \in A_i} \exp[\lambda \beta_{0a} + \sum_{k \in K_a} \lambda \beta_{ka} x_{ka}]} \right) \quad (6)$$

where:

LL = log-likelihood

The log-likelihood function needs to be maximized. Thus, must search for the values of  $\beta$  that maximize the log-likelihood function. The set of utility function parameters to be estimated is represented by  $\hat{\beta}$  as seen in Formula 7:

$$\hat{\beta} = f(\beta | \Theta) \quad (7)$$

where:

$\Theta$  is the true value of the parameters.

To find the maximum likelihood estimate of  $\beta$ , we equate the derivative of the log-likelihood function to zero and solve for  $\beta$  using methods such as Newton Raphson or Scoring found in a work by Maddala [13]. Maddala also explains that if Newton Raphson method is used, point estimates for these are found using Formula 8:

$$\beta_1 = \beta_0 + [I(\beta_0)]^{-1} S(\beta_0) \quad \text{until} \quad |\hat{\beta}_{n+1} - \hat{\beta}_n| < \epsilon \quad (8)$$

where:

$I(\beta)$  = the information matrix or the matrix of the second derivatives of the log-likelihood function for every individual observation  $i$  which is positive definite for each iteration as seen in Formula 9 and 10:

$$I(\beta) = E \left( -\frac{\partial^2 \log LL}{\partial \beta \partial \beta} \right)$$

(9)

$$= \sum_{i=1}^n \frac{\exp(\beta x_i)}{[1 + \exp(\beta x_i)]^2} x_i x_i'$$

(10)

and  $S(\beta)$  = the standard errors or the matrix of first derivatives of the log-likelihood function for every individual observation  $I$  as seen in Formula 11:

$$S(\beta) = \sum_{i=1}^n \frac{\exp(\beta x_i)}{1 + \exp(\beta x_i)} x_i$$

(11)

The standard errors for these estimates can be found by determining the co-variances using the inverse of the second derivatives matrix  $[I(\beta)]^{-1}$ . This is the basic estimation process. The basic estimation process does not assume any variations existing in  $i$ 's,  $a$ 's, or  $ia$ 's. The purpose of this paper is to show what will happen to the  $LL$  if there is in fact a variation in  $i$ 's,  $a$ 's, or  $ia$ 's. The following three sections respectively explain what happens to  $LL$  if such variations exist. Section 3.2 regarding the variations in  $i$ 's. In basic estimations, it is assumed the survey sample observations could be used to draw conclusions about the entire population. However, this is not a valid assumption and the  $LL$  needs to be modified. Section 3.3 addresses the variations in  $a$ 's if they exist. In the basic estimations, it is assumed that the alternatives, i.e.  $a$ 's, have equal probability of appearance in each game or observation.

However, in many SP surveys only a sub-sample of the alternatives is available to the respondents. For example, consider a SP study that the popularity of six different modes of travel are to be examined. However, the SP games are designed such that each respondent gets to choose five random cards in each game that could include all five alternatives or five different variations of only one of those modes. Thus, an uncertainty is introduced in

the frequency of alternatives' availability. Section 3.4 explains possible variations in the  $ia$ 's where a correction method is explained for many SP cases where each individual plays the game more than once and thus a correlation exists among the responses and those responses cannot be treated independent and irrelevant observations.

### 3.2 Adjustment Factor Against the Entire Population (Theoretical Concept)

In order for the survey data to be an appropriate representative of a population, the results can be adjusted against the entire population using a weight factor [14]. The log likelihood for a weighted estimator is equivalent to that for the original estimator, except that each observation is weighted against the entire population. The following Formula 12 and 13 show how the likelihood function is weighted with expansion factor weights  $\omega$ . The formula is implemented in software using a weight factor  $\omega$  for each observation [12]:

$$LL = \ln(L) = \sum_{i \in I} \omega_i \ln(P(a_i^*)) = \sum_{i \in I} \omega_i \ln \left( \frac{\exp[\lambda \beta_{0a} + \sum_{k \in K_a} \lambda \beta_{ka} x_{ka}]}{\sum_{a \in A_i} \exp[\lambda \beta_{0a} + \sum_{k \in K_a} \lambda \beta_{ka} x_{ka}]} \right)$$

(12)

$$\omega_i = \frac{\text{Population Share}}{\text{Sample Share}} \quad (13)$$

where:

$\omega_i$  = the ratio of population shares for each observation. This adjustment will be called weight adjustment throughout this text.

### 3.3 Adjustment for Uncertainty in Frequency of Alternatives' Availability (Theoretical Concept)

In the SP survey of this project we present the interviewees with five individual cards during each observation. There are six alternatives in total. Two of the cards are bus alternatives at all times and the remaining three could be any random combination of the modes taxi, ride with friend already driving, ride with friend not already driving, ride with relation already driving, and ride with relation not already driving. There is an uncertainty in the availability of alternatives in each observation. One of the underlying assumptions in maximum likelihood estimation process is the

equal probability of appearance of all the alternatives at all times in which case the following Formula 14 holds [15]:

$$P(i^*) = \frac{e^{[V_i^* + \varepsilon_i^*]}}{\sum_{i \in I} e^{[V_i + \varepsilon_i]}} \quad (14)$$

This equation is valid under the assumption that the entire set of  $V_i$  is always available for all  $i$ . However, there is a need to correct for some of the alternatives being unavailable in some cases. To do so we need to include an adjustment factor for all alternatives in the utility functions. More information regarding this adjustment can be found in [16]. In the case study herein, we can use the probability of appearance of a card in a draw and calculate the correction factor based on that probability. The following section illustrates the procedure to calculate such correction factor for inclusion in the utility functions:

where:

- $i \in C$  = unique alternatives from the full set
- $i \in D \subset C$  = unique alternatives from the sample
- $q(i)$  = selection probability (prob. to be drawn)
- $n_i$  = selection frequency in the sample
- $N$  = sample size
- $V_i$  = utility of a choice alternative
- $P(i)$  = choice probability

The choice frequencies of each of the alternatives are summed over the sample size as seen in Formula 15.

$$\sum_{i \in D} n_i = N. \quad (15)$$

In Sample D, for all alternatives from 1 to N, the choice probability with sampling correction factors is seen in Formula 16:

$$P(i) = \frac{\exp[V_i + \ln\left(\frac{n_i}{N \times q(i)}\right)]}{\sum_{j \in D} \exp[V_j + \ln\left(\frac{n_j}{N \times q(j)}\right)]} = \frac{\left(\frac{n_i}{N \times q(i)}\right) \times \exp(V_i)}{\sum_{j \in D} \left[\left(\frac{n_j}{N \times q(j)}\right) \times \exp(V_j)\right]} \quad (16)$$

with cancellation of  $N$  which is a fixed number we have Formula 17:

$$P(i) = \frac{\exp[V_i + \ln\left(\frac{n_i}{q(i)}\right)]}{\sum_{j \in D} \exp[V_j + \ln\left(\frac{n_j}{q(j)}\right)]} = \frac{\left(\frac{n_i}{q(i)}\right) \times \exp(V_i)}{\sum_{j \in D} \left[\left(\frac{n_j}{q(j)}\right) \times \exp(V_j)\right]} \quad (17)$$

The utility correction factor in Formula 16 is assumed to be  $\ln\left(\frac{n_i}{N \times q(i)}\right)$ ; However, the correction factor assumed in Formula 17 is  $\ln\left(\frac{n_i}{q(i)}\right)$ . The results of both formulas are the same and thus the simple form of correction factor presented in Formula 17 is used. In the case where a log-sum of this selection model needs to be applied in an upper level choice model, then the log-sum needs to be calculated as Formula 17 denominator as seen in Formula 18:

$$LL^1 = \ln \left\{ \sum_{j \in D} \exp[V_j + \ln\left(\frac{n_j}{N \times q(j)}\right)] \right\} = \ln \left( \sum_{j \in D} \left(\frac{n_j}{N \times q(j)}\right) \times \exp(V_j) \right) \quad (18)$$

In the case that Formula 17 is used for the choice probability correction, then the log-sum in Formula 19 would be:

$$LL^2 = \ln \left\{ \sum_{j \in D} \exp[V_j + \ln\left(\frac{n_j}{q(j)}\right)] \right\} = \ln \left( \sum_{j \in D} \left(\frac{n_j}{q(j)}\right) \times \exp(V_j) \right). \quad (19)$$

Now log-sum Formula 19 that was calculated using Formula 17 needs to be scaled in order to reproduce the value of Formula 18 which was calculated using formula 16 as can be seen in Formula 20:

$$LL^2 = LL^1 - \ln(N). \quad (20)$$

Therefore, the two ways to apply corrections for choice model and log-sum calculations are:

1. Using Formula 16 for utility correction factors, then using the log-sum directly from the denominator of the Formula 16
2. Using Formula 17 for utility correction factors, then scaling the log-sum from the denominator by Formula 20

In case we assume the entire set of choice frequencies are one meaning that ( $n_j = 1$ ) and with all selection probabilities being equal  $q(j) = q = \frac{1}{R}$  the Formula 18 will simply be:

$$LL^1 = \ln \left( \sum_{j \in D} \left( \frac{R}{N} \right) \times \exp(V_j) \right) = \ln \left( \sum_{j \in D} \exp(V_j) \right) + \ln \left( \frac{R}{N} \right) \quad (21)$$

where:

- R = size of the full set
- N = sample size
- $V_j$  = utility of a choice alternative

Formula 21 can be used for basic estimations when the original log-sum was calculated without correction factors and random sampling without replacement. Then the log-sum needs an expansion factor equal to the full set size divided by the sample size which is called frequency adjustment hereafter.

### 3.4 Adjusting for Multiple Repeated Observations (Theoretical Concept)

In many surveys, where possible, multiple observations are obtained from each respondent in an attempt to get more information at lower marginal cost. This is especially true with SP experiments, where it would seem that the flexibility of the process and the relative ease with which different hypothetical situations can be described invites multiple observations [17].

In the typical SP survey, the respondent is asked to participate in a game-like process, where the same choice situation is considered multiple times, each time with a different set of alternatives to be ranked in order of preference [18]. Asking one person to play 10 games is not the same as asking 10 people to each play one game. The nature of the information obtained will be different.

In the first case, there is more information about one person and in the second case there is information about more people. In the first case, because it is the same person, all other things being equal, the displayed behavior will tend to be much more similar and thus the data more correlated. Intuitively, there will be less variation and thus less information about the relevant choice behavior for the population

from a sample of 10 observations all from the same person than from a sample of 10 observations from each of 10 different people.

In the development of the standard single-level logit model and the estimation processes that work with it, it is assumed that the errors in the utility values are identically and independently distributed across all the alternatives for all the observations. This assumption may be reasonable when there is just one observation from each respondent, but it seems much less reasonable when there are multiple observations from each respondent.

Clearly this cannot be valid in the case where there are multiple observations per respondent. A well-recognized study [19] has clearly explained about an SP experiment with repeated observations in which the respondents are offered a choice between coach and train. Attributes such as "In-Vehicle-Time" and "Wait-Time" were considered for each mode of travel.

The respondents were asked to go through the survey more than one time which can create an auto-correlation problem in the data collected as collecting say six responses from one person is not equal to collecting one response from six individuals [17]. In the case of the SP survey here in, we ask the respondents to repeat the survey six times in total. This implies that there are correlations among the six different sets of responses collected from the interviewees. Thus, we can no longer assume that all our observations are identically and independently distributed.

This is important because there are some fundamental assumptions in the maximum likelihood estimation process, which requires the IIA property to hold at all times. Thus, the error terms of the utility functions which were assumed to be independently and identically distributed, need to be corrected by a factor that accounts for the interdependency of observations collected from each interviewee.

One method is to use the Jackknife estimate of standard, introduced by Tukey (1958). Jackknife, described by [20] gives nonparametric estimation of variances. The Jackknife variance described by [21] can be estimated in five steps:

1. Let  $\varphi$  represent the sample, which consists of N observations. Partition  $\varphi$  as  $\varphi = \{ \varphi_1, \varphi_2, \dots, \varphi_G \}$  where G is the

number of groups, each of size  $m$ , such that  $G \times m = N$ .

2. Let  $\hat{\beta}$  represent the estimator obtained using all the data (i.e.  $j$ ). Let  $\hat{\beta}_{-i}$  represent the estimator obtained by omitting  $\varphi_i$ , for  $i = 1, 2, \dots, G$ , then
3. Calculate the pseudo values as seen in Formula 22:

$$\beta_i^* = G\hat{\beta} - (G - 1)\hat{\beta}_{-i}, \text{ for } i = 1, 2, \dots, G. \tag{22}$$

these  $\beta_i^*$  can be treated like  $G$  observations of  $\hat{\beta}$  (though not independent).

4. Calculate the sample average of the pseudo values as in Formula 23:

$$\bar{\beta}^* = \frac{1}{G} \sum_{i=1}^G \beta_i^*. \tag{23}$$

5. Estimate the variance of  $\hat{\beta}$  as seen in Formula 24:

$$Var(\hat{\beta}) = \frac{1}{(G-1)G} \sum_{i=1}^G (\beta_i^* - \bar{\beta}^*)(\beta_i^* - \bar{\beta}^*)'. \tag{24}$$

For each coefficient  $k$  in the original model the correct t-statistic, based on the jackknife estimates of the variance, are calculated as seen in Formula 25:

$$\frac{\hat{\beta}_k}{\sqrt{Var(\hat{\beta})_{kk}}} \tag{25}$$

#### 4 Method

The data used for this study is collected from a SP survey where respondents are presented with a deck of cards on which different modes of travel along with their specific attributes are printed. Each respondent will draw five random cards from a deck of such cards and is asked to rank and rate the cards in order of preference with each card representing a specific mode. The survey design was based on a hypotheticalal scenario in which the respondents were to travel to a meeting while

they did not have the option to drive alone. The respondents had to choose among different alternative travel options namely bus, taxi, or ride as a passenger with a friend or a relative. Six different scenarios printed on cards (1008 cards in total) were Bus (576 cards), Taxi (96 cards), Ride with a friend who is already driving (112 cards), Ride with a friend who is not already driving (56 cards), Ride with a family member living in the household who is already driving (112 cards), and Ride with a family member living in the household who is not already driving (56 cards). For each scenario, there are different attributes selected. Each attribute has different states, levels, and values. The values will be randomly chosen for different cards which yield to a pile of 300 cards with each card having different attribute values than the other. See Figure 2 for sample survey cards. The process of ranking and rating five cards is repeated six times for each respondent and the results are recorded. The selection of mode attributes has been based on the review of literature. The attributes are considered to be some of the major attributes of a travel mode that could influence one's decision on selection of that mode. Some of the alternative (mode) specific attributes are listed in Table 1. A total of 717 suitable interviews were selected and 4212 runs of ordering and rating cards will be used in the analyzing of factors. The utility function for each mode of travel is then defined as a combination of the attributes on the cards and their respective coefficients. The Formula 26 below is a sample utility function that will be used in the estimation process:

$$U_{Bus} = [BWalkorig \times x_{BWalkorig}] + [BWalkDest \times x_{BWalkDest}] + [BScheMis \times x_{BScheMis}] + [BRide \times x_{BRide}] + [BFare \times x_{BFare}] \tag{26}$$

Figure 3 is a graphical representation of the resulting sample's age and gender distribution with the left column representing males and the right column representing females. All 717 interviews had gender information recorded. 425 (59.27%) male and 292 (40.73%) female respondents participated in the survey. The distribution of income for the obtained sample is shown in Figure 4.

## 5 Results

The no tree structure was used as a general case for the study herein which can be represented by Figure 5. It means that it was assumed that all the six modes of travel have equal elasticity. Each stem of this structure is connected to a common "traveller" or "root" point which means that upon any changes in the availability of one mode, all the other modes will be affected equally. However, this is not always the case especially when there are alternatives that are more similar such as the friends or HH relation alternatives. The next sections describe the application of the three adjustments on the raw data considering no tree structure along with one case where a full adjusted nested tree structure as seen in Figure 6 is chosen. The impact of applying each of those adjustments on the coefficient estimates have been summarized in Table 2 for convenient comparisons. The t-statistic values for specific pairs of coefficients after the application of adjustment factors have been shown in Tables 3 to 7.

### 5.1 Results of Naive Estimations without any Adjustments (Empirical Example)

The Original Results column of Table 2 represents the estimated values of the coefficients and their t-ratios for the case where no adjustment was applied to the original data. Table 3 illustrates the values of t-statistics for different pairs of coefficients with no adjustments.

### 5.2 Adjustment Factor Against the Entire Population (Empirical Example)

The experimental results obtained from the surveys only represent the survey sample. In order to be able to draw conclusions regarding a city or a region, these data need to be extrapolated to a larger population [14].

This is achievable by determining the age, income, and gender category of each respondent and then using the statistics tables provided by Statistics Canada found in [22], [23], [24], and [25] to calculate an expansion factor for each observation [26]. One method to calculate the expansion factor is by applying an Iterative Proportional Fitting IPF method [27]. See the weight section in Tables 2 and 4.

### 5.3 Adjustment Factor for Uncertainty in Frequency of Alternatives' Availability (Empirical Example)

In a work done by [16], the process of adjusting for uncertainty in frequency of alternatives' availability is referred to as limiting the number of alternatives. See Table 2 and 5 frequency section.

### 5.4 Adjusting for Multiple Repeated Observations per respondent (Empirical Example)

If the problem of autocorrelation is ignored, the t-ratios calculated for estimated parameters in the model tend to be larger than what they are [1]. Thus, it cannot be concluded with certainty that our parameter estimations are truly statistically significant in cases where the actual value of t-ratios is less than the 2.0 acceptance limit. In this project, the Jackknife process was undertaken using the latest version of the estimation software ALOGIT 4.2. See the Jackknife section presented in both Table 2 and Table 6 for results of this adjustment.

### 5.5 Results of Application of All Three Adjustments with no Tree

Table 7 represents the t-statistics values for pairs of coefficients after the application of all three adjustment factors on the original data using a no nested tree as seen in Figure 5.

### 5.6 Results of Application of All Three Adjustments with a Full Adjusted Tree

Table 8 represents the t-statistics values for pairs of coefficients after the application of all three adjustment factors on the original data using a full adjusted nested tree structure as seen in Figure 6.

## 6 Analysis

The following sections summarize the results of application of adjustments. First, changes in the signs of the estimated parameters due to the adjustments have been discussed. The values of the t-ratios calculated by the logit software is then analyzed.

Next, changes in the values of the estimated parameters have been investigated. Finally, the values of the t-statistics between different pairs of parameters before and after the adjustments have been discussed.

### 6.1 Estimated Parameter Sign Changes



As it was seen in Table 2, the signs for the estimated parameters before and after adjustments stays the same for all parameters except for two constants with the first being:

*RConstant* which is negative in two occasions (weight and Jackknife/weight shown in boxes in Table 9 and positive in all other cases).

*FConstant*: negative in three cases (original, Jackknife, and weight) and positive in all other cases as seen in Table 10.

## 6.2 Changes in Absolute Value of T-Ratios

The values of the t-ratios for the original case and the all factors case have been compared here. In most cases, there is a drop in the t-ratio values most likely due the Jackknife adjustments. The Jackknife process as explained in Section 3.4 reduces the errors in estimates and thus a reduction in the t-ratios is expected. In all parameters except a few explained here, a t-ratio drop is visible. However, the drop is not big enough to make the t-ratio statistically insignificant. Table 11 shows the results for all alternatives with the following observations:

1. *RConstant* t-ratio which is non-significant except whenever frequency adjustment is used.
2. *FConstant* t-ratio: t-ratio significant in the original case and stays significant for all other cases except in the weight case as well as the Jackknife plus weight case.
3. *DConstant* t-ratio: t-ratio significant and very high in the original case and t-ratio dropped to be marginally significant after all three adjustments were applied.
4. *HConstant* t-ratio: greater than 2.0 in one case only (All Factors) and less than 2.0 in all other cases.
5. *TRide* parameter t-ratio: insignificant whenever Jackknife and weight are both applied and significant in all other cases.

## 6.3 Estimated Parameter Values

A look at Table 2 as was shown previously indicates that the estimated values of the coefficients in all cases have not been subject to a major change after applying different adjustments. The values for the parameters in the base case have been compared the values

of the parameters obtained after each adjustment in Table 12. This has been done by calculating the t-statistics for the changes in parameter values. The values of estimated parameters are very close in almost all cases and the t-statistics of the differences are not statistically significant. The results show that in the case of the Jackknife corrections no difference is seen in the value of coefficient estimates as expected and explained by [15]. Although the results are very similar, but they are in fact different after application of each adjustment factor. Thus, one might be interested to try applying these adjustment factors for different projects with different data sets to ensure that the difference in the estimated values are not statistically significant.

## 6.4 Estimated Constant Values

Unlike parameters, the estimated values of the constants have had a significant change in almost all adjustment cases except for the Jackknife case. To figure out if those changes are statistically significant, Table 13 is provided which represents the t-statistic values calculated for the changes in the values of the constants after applying each adjustment. To do this, the values for the constants in the base case have been compared to the values of the constants obtained after application of different adjustments. The changes in the values of the constants are mostly statistically significant at 95% confidence interval in the cases of weight and Jackknife plus weight (except for *FConstant*). This might be of interest for projects where decisions are made based upon the values of the constants.

## 6.5 T-Statistic Values Between Pairs of Parameters

This section discusses the results shown in Tables 3 through 7. In those tables a t-statistic value was calculated for different pairs of parameters. Some changes could be seen in the values of t-statistics after each adjustment was applied on the original data.

1. Base Case vs Weighted Data:

In Table 4 there were nine pairs of parameters that the difference in their values were found to be insignificant while thought to be significant if only the original results from Table 3 had been

used for analysis. On the other hand, Table 4 contains four pairs of parameters with statistically significant differences in their values that were previously found to be insignificant in the original case. In utility functions when two variables are NOT statistically significant from each other they need to be combined and better not be kept as two separate variables in the. If only the original results were used, some of these pairs of parameters would have been kept separate or combined which could influence the results of the entire estimations.

2. Base Case vs. Frequency Adjustment Factor Results: The results after the frequency adjustments in Table 4 show that the values of all the parameters except for the constants are the same as the base case. The reason could be due to the fact that for the specific data set used in this study the frequency adjustment factors calculated to be added to the utility functions were very small mostly below 1.5. After applying Frequency adjustments which accounts for this inequality in the probability of alternatives' availability, the values of the alternative specific constants have all changed to a positive value meaning they are greater than the reference point zero for *BConstant*. This shows that the bias has been removed and all the other alternatives have higher desirability compared to the Bus alternative.
3. Base Case vs. Multiple Repeated Observations: The significance of t-statistics before and after Jackknife adjustments remains the same for almost all pairs of parameters as seen in Table 6. This is expected as Jackknife process will affect the t-ratios and errors in estimations and not the values of the parameter.
4. Base Case vs. All Adjustment: Table 7 shows that many pair of parameters have statistically insignificant differences in values after all three adjustments have been applied. However, many of the t-statistics are marginally significant which could be due to the noise in the Jackknife process. Thus, they might

be still significant but as the two other adjustments (weight and frequency) are also used, then it is expected to see such discrepancies. If the estimations are done for many times and the average t-statistics are used, these marginally significant t-statistics might show to be significant.

## 6.6 The Value of Time (VOT)

The VOTs calculated as seen in Table 14 for the bus alternative is reasonably close to the previous findings in the literature which was around 4.0 dollar per hour [28]. The VOTs calculated using DRide and HRide are close to the value of \$18 found previously for the City of Calgary [26]. It can be seen that the VOTs found using the RRide and FRide are at least half the values calculated using DRide and HRide. This is another indication of the the added discomfort felt when extra travel time are imposed on the driver. Large t-ratios for the extra ride times and the schedule mismatch between the driver and the passenger for the R and F modes (RRideExtra, FRideExtra, RSchedMisF, and FSchedMisR) can be observed.

The estimated values of these factors might be subject to some influences and the overall results might be mixed. However, it can be concluded that the parameters that represent the sensitivity of the traveler to the extra ride time imposed on a driver as well as the schedule mismatch of the driver play important roles in decision to ask for a ride. The values show that people feel the discomfort when they are imposing a favour on someone. This is a direct empirical evidence of altruism or maybe it's the discomfort that they are seeking to avoid. The story behind not deciding to phone someone to ask for a ride is perhaps not because the traveller is feeling the pain of his or her own time but because he/she is feeling the awkwardness concerning the driver's time. It means that the decision to rideshare is an internal process. What is clear from the results here is that there is increasing disutility associated with the driver's extra time and this could be substantial enough to tip the balance away from ride sharing.

## 6.7 Adjusted Nested Tree Structure

All of the above results were with respect to a basic non-nested tree structure. Since ALOGIT was used herein, then some adjustments

regarding the nesting parameters needs to be considered when there is a nesting structure with different levels and branches [29]. Table 8 provided an example representing the t-statistics values with all three SP adjustments used under an adjusted nested tree structure. Even though the results show almost no differences, in certain adjusted nesting structures as seen in [30], the application of all three SP adjustments explained here in Sections 3.2 to 3.4, revealed to have statistically significant impacts on the value of t-statistics and t-ratios while improving the fit of the model. In practical cases this outcome can act as a turning point in policy and decision-making processes. This is because if policy decisions are to be based on the results of such studies, attention needs to be paid to the types of the tree structures used in the study.

## 7 CONCLUSIONS

The weight correction factor did not change the value of coefficients or constants by a significant amount. However, many large t-statistic values between pairs of parameters in the base case showed a significant decrease after the weight adjustments were applied. The adjustment for uncertainty in frequency of alternatives' availability resulted in increases in the values of some of the constants and therefore their sign changed from negative to positive. Those cases were related to the alternatives that had lower number of cards compared to other alternatives in the full set of cards. The Jackknife process here showed to have no effect on the estimated values of parameters, the constants, nor the t-ratios.

However, depending on the nature of the data, some projects might be faced with a situation where their t-ratio values drop below 2.0 and forced to reject their hypothesis to use the corresponding coefficients in their model [18]. There were no statistically significant changes seen in the estimated values of parameters after any of the adjustment factors were applied. In almost all cases, the changes in the values of the constants found to be statistically significant at 95% confidence interval after application of the weight as well as the Jackknife/ weight adjustments. Some of the t-statistics between different pairs of parameters, turned out to become statistically significant (or vice versa) after the weight as well as the frequency adjustments. Most t-

ratios with a very large value in the original case showed a decrease to a reasonable value after the Jackknife process.

Here, using all three adjustments separately or in combination did not result in statistically significant changes in the value of parameters, t-ratios, nor the fit of the model using a basic tree structure. However, even though the adjustments, while necessary, didn't change the empirical results, caution should be exercised in their interpretation and the conclusions may not be generalized. This is because, in another work by the same first author, using the same data set, when different nesting structures among groups of alternatives were used (Hassanvand, 2012), the final results were found to show statistically significant differences.

The said work showed how under some specific adjusted nested tree structures, the absence of one or two of the adjustments explained herein had statistically significant effects on the t-statistics results and the statistical fit of the model. Thus, under certain nesting structures and different data sets, leaving out even one of the adjustments discussed herein can result in less accurate outcomes and researchers should expect sizable differences as a result of the adjustments which would require testing various nesting structures. Considering various cases and nesting structures then becomes of utmost importance and gains practical significance to policy makers if decisions are to be made based on the conclusions drawn from SP studies. Overall in the work herein most of the statistically significant changes have been seen in the values of the constants. This might be of interest for policy makers as many transportation related decisions are based on the values of the constants. Thus, these adjustments could be useful for projects that their results are important for policy makers.

In particular, the effect of frequency adjustments on constants might be important in some practical planning projects. When considering the Jackknife process, there is a possibility of a drop in the value of the parameters' t-ratios resulting in those parameters to be viewed as non-significance. In such a case a change in utility functions might be needed. In general, there are some difficulties when undertaking research such as

one described herein. One is that the calculation of factors in this project was done using Excel® spreadsheets. In some cases, use of macros saved a lot of time in the calculation process.

Also, in the process of literature review, there were not many studies that focused on the nature and use of all three adjustments together when SP data is used. This paper has done a detailed look at SP data's possible limitations and different avenues regarding using naïve analysis were investigated here. This work is a review of the collection of adjustment factors for use by those who are dealing with SP data. Instead of Jackknife, one could use the extended sandwich estimator method for more accurate results [15]. Also, when applying all adjustments, it is useful to do the estimations few times to realize of any consistencies are seen in the final results. This will ensure that the effect of randomness in the Jackknife on final results is not ignored.

## 8 ACKNOWLEDGEMENTS

Dr. John Bowman and professor Andrew Daly's insight, guidance, and help through electronic email messaging regarding concepts included in this paper is highly appreciated.

## 9 REFERENCES

- [1] Bates, J.J. 1988. Economic Issues in Stated Preference Analysis. *Journal of Transport Economics and Policy*, 22(1), 9-69.
- [2] Ghaffari, M. 2017. Factors Influencing the Use of Autonomous and Shared Autonomous Vehicles. University of Calgary, Calgary, Alberta CANADA (Master's thesis).
- [3] Wardman, M. and Bristow, A. L. 2004. Traffic related noise and air quality valuations: evidence from stated preference and residential choice models. *Transportation research D* 9: 1-27.
- [4] Wardman, M., Bristow, A. L., and Arsenio, E. 2005. Applying Stated Preference Methods to the Valuation of Noise: Some Lessons to Date. Paper prepared for the Congress and Exposition on Noise Control Engineering, 7-10 August, Rio de Janeiro, Brazil.
- [5] Bai, Y. 2014. Modelling Riders' Behavioural Responses to Real-Time Information at LRT

Stations in Calgary, Alberta. University of Calgary, Calgary, Alberta CANADA (Master's thesis).

[6] Hossain, M. S. 2011. A Stated Preference Examination of Sensitivities to Transit-Related Walking and Waiting in Calgary. University of Calgary, Calgary, Alberta CANADA (Master's thesis).

[7] Cherchi, E., Ortúzar, J. de D. 2011. On the use of mixed RP/SP models in prediction: accounting for random taste heterogeneity. *Transp. Sci.* 45(1), 98-108.

[8] Bliemer, M. C. J., Rose, J. M., & Chorus, C. G. 2017. Detecting dominance in stated choice data and accounting for dominance-based scale differences in logit models. *Transportation Research. Part B: Methodological*, 102, 83-104. <https://doi.org/10.1016/j.trb.2017.05.005>.

[9] Henn, V., Ottomanelli, M. 2005. Handling Uncertainty in route choice models: From probabilistic to possibilistic approaches. *European Journal of Operational Research* 175: pp. 1526- 1538

[10] Vitetta A., 2016. A quantum utility model for route choice in transport systems. *Travel Behaviour and Society*, N. 3, Pag. 29-37

[11] Ben-Akiva, M. and S.R. Lerman. 1985. *Discrete Choice Analysis: Theory and Application to Travel Demand*, Cambridge, MA: MIT Press.

[12] Garrow, L. A. 2010. *Discrete choice modelling and air travel demand: Theory and applications*. England: Ashgate Publishing Limited.

[13] Maddala, G. S. 1983. *Limited-dependent and qualitative variables in econometrics*. Cambridge Uni Press.

[14] Bhat, C.R., Schofer, J.L., Koppelman, F.S. and Bautch, R.C. 1993. Driver Recruitability for Advanced Traveler Information System Experiments. *Transportation Research Part C*, 1, 4, 265-274.

- [15] Daly, A. and Hess, S. 2010. Simple Approaches for Random Utility Modelling with Panel data. presented at the 12<sup>th</sup> WCTR, Lisbon, Portugal.
- [16] McFadden, D. 1984. Econometric analysis of qualitative response models. In Griliches, Z. & Intriligator, M. eds., Handbook of Econometrics, North Holland, 1385-1457.
- [17] Ouwersloot, H. and Rietveld, P. 1996. Stated choice experiments with repeated observations, Journal of Transportation Economics and Policy, 30, 203-212.
- [18] Hess, S. and Rose, J.M. 2006. Effects of distributional assumptions on conditional estimates from Mixed Logit models, ETH Working Paper, Arbeitsberichte Verkehrs- und Raumplanung, 423, IVT, ETH Zurich.
- [19] Wardman, M. 1988. A Comparison of Revealed Preference and Stated Preference Models of Travel Behaviour. Journal of Transport Economics and Policy, 22(1), 71-91.
- [20] Miller, R. 1974. The Jackknife: A Review. Biometrika, 61, 1-14.
- [21] Labbé, M., Laporte, G., Tanczos, K., and Toint, P. (Eds.). 2013. Operations research and decision aid methodologies in traffic and transportation management (Vol. 166). Springer Science & Business Media.
- [22] Statistics Canada. 2008. Family Income Groups (22) in Constant (2005) Dollars, Economic Family Structure (9) and Economic Family Size (5) for the Economic Families in Private Households of Canada, Provinces, Territories, Census Metropolitan Areas and Census Agglomerations, 2006 Census, 20% Data (table). Topic Based Tabulations: Income and Earnings. Statistics Canada catalogue no. 97-563-XCB2006020. Ottawa. May 1. <http://www12.statcan.gc.ca/census-recensement/2006/dp-pd/tbt/index-eng.cfm> (accessed June 14,2011).
- [23] Statistics Canada. 2008. Household Income (7) and Household Size (4) for the Private Households of Canada, Provinces, Territories, Census Metropolitan Areas and Census Agglomerations, 2006 Census, 20% Data (table). Topic Based Tabulations: Income and Earnings. Statistics Canada catalogue no. 97-563-XCB2006051. Ottawa. May 1. <http://www12.statcan.gc.ca/census-recensement/2006/dp-pd/tbt/index-eng.cfm> (accessed June 14,2011)
- [24] Statistics Canada. 2008. After-tax Household Income Groups (21) and Household Type (11) for the Private Households of Canada, Provinces, Territories, Census Metropolitan Areas and Census Agglomerations, 2006 Census, 20% Data (table). Topic Based Tabulations: Income and Earnings. Statistics Canada catalogue no. 97-563-XCB2006046. Ottawa. September 30. <http://www12.statcan.gc.ca/census-recensement/2006/dp-pd/tbt/index-eng.cfm> (accessed June 14,2011).
- [25] Statistics Canada. 2008. Household Income Groups (24) in Constant 2005 Dollars and Household Type and Number of Persons 65 Years and Over (15) for the Private Households of Canada, Provinces, Territories, Census Metropolitan Areas and Census Agglomerations, 2006 Census, 20% Data. Topic Based Tabulations: Income Earnings. Statistics Canada catalogue no. 97-563-XCB2006047. Ottawa. September 30. <http://www12.statcan.gc.ca/census-recensement/2006/dp-pd/tbt/index-eng.cfm> (accessed June 14,2011).
- [26] Hunt, J. D. 2008, February 19. ENCI 707 course. Lecture, Topic: "Logit Choice Model." Room ENF 330, Faculty of Civil Engineering, University of Calgary, Calgary, Alberta.
- [27] Norman, P. 1999. Putting Iterative Proportional Fitting on the Researcher's Desk, forthcoming working paper, School of Geography, University of Leeds, Leeds.
- [28] McMillan, J.D.P. 1996. A Stated Preference Investigation of Commuters Attitudes Towards Carpooling in Calgary and Edmonton. MSc Thesis, University of Calgary, Calgary AB, Canada.
- [29] Daly, A. J. 2017. ALOGIT Users' Guide, Version 4.2: January 2007. Hague Consulting Group, The Hague, The Netherlands.

[30] Hassanvand, M. 2012. Factors Affecting Utility of Passenger Modes in Calgary. University of Calgary, Calgary, Alberta CANADA (Master's thesis).

[31] CBC, Robson Fletcher. 2018. What the daily grind of downtown commuting tells us about Calgary. Available from: <https://www.cbc.ca/news/canada/calgary/calgary-2018-downtown-cordon-count-cars-bikes-transit-1.4876641>

**Bus**

Walk Distance to Stop	Ride Time Stop to Stop	Bus Fare one-way	Walk Distance from Stop	Arrive Before or Leave After Meeting
<b>7</b> blocks	<b>45</b> minutes	<b>2.50</b> \$	<b>7</b> blocks	<b>25</b> minutes

2 B Z D

**10 FIGURES AND TABLES**

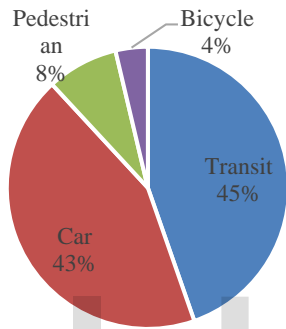


Figure 1: City of Calgary Mode Share for Downtown Rush Hour 7:30 to 8:30 AM in 2018 [31]

**Taxi**

Ride Time Door to Door	Taxi Fare one-way	Arrive Before or Leave After Meeting
<b>15</b> minutes	<b>18.50</b> \$	<b>25</b> minutes

2 T A L

Figure 2: Sample Survey Cards

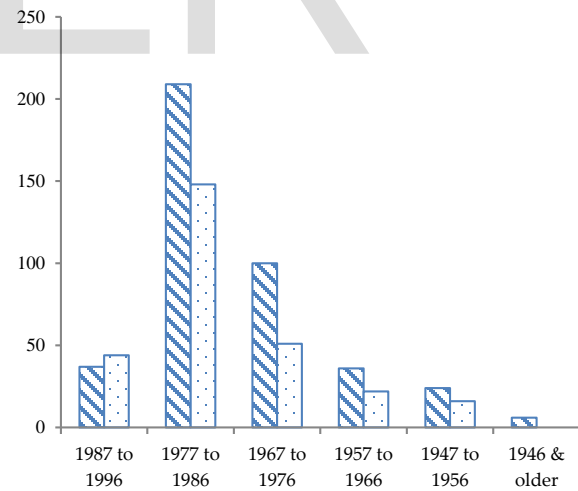


Figure 3: Distribution of Age and Gender (male: left column, female: right column)

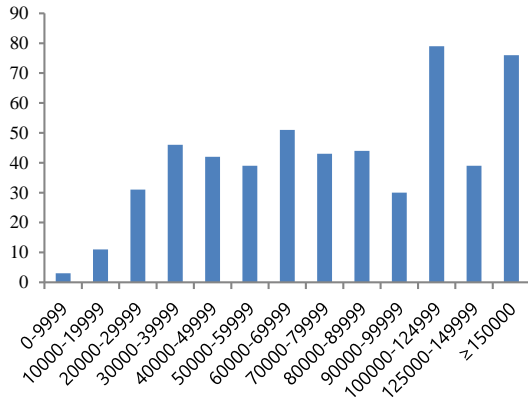


Figure 4: Income Distribution of Sample

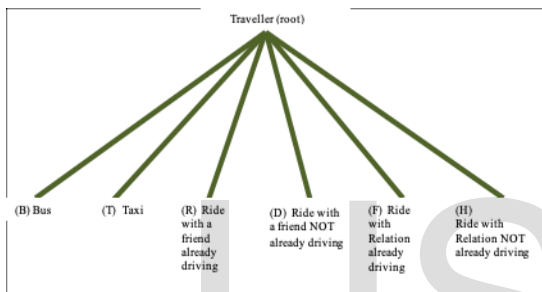


Figure 5: No Tree Structure

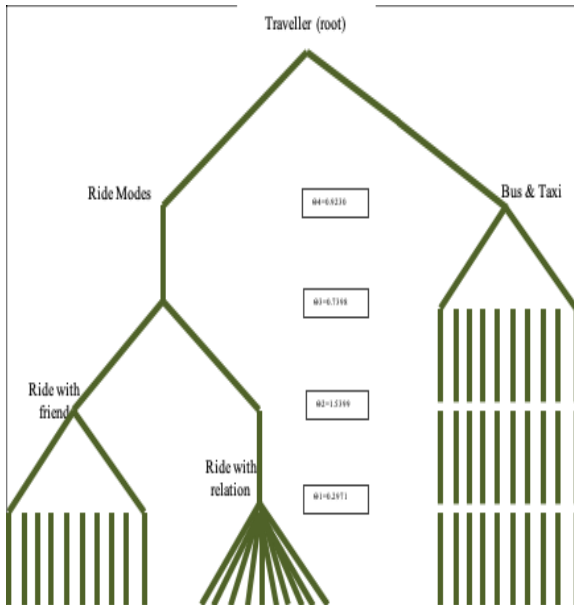


Figure 6: A Full Adjusted Nested Tree

Table 1: Selected Modes of Travel for Surveys

Mode(abbreviation) Parameters	Description of Parameter
<b>1 - BUS (B)</b>	
BWalkorig	Bus walk distance from stop
BRide	Bus ride time
BFare	Bus fare
BWalkDest	Bus walk distance from destination
BScheMis	Bus schedule mismatch
<b>2 - Taxi (T)</b>	
TRide	Taxi ride time
TFare	Taxi fare
TSchedMis	Taxi schedule mismatch
<b>3 - Ride with friend already driving (R)</b>	
RRideExtraF	R Mode extra ride time f
RRide	R Mode ride time
RSchedMisF	R Mode schedule mismatch
RSchedMis	R Mode schedule mismatch
<b>4 - Ride with friend NOT already driving (D)</b>	
DRide	D Mode ride time
DSchedMis	D Mode schedule mismatch
<b>5 - Ride with HH relation already driving (F)</b>	
FRideExtraR	F Mode extra ride time f
FRide	F Mode ride time
FSchedMisR	F Mode schedule mismatch
FSchedMis	F Mode schedule mismatch
<b>6 - Ride with HH relation NOT already driving (H)</b>	
HRide	H Mode ride time
HSchedMis	H Mode schedule mismatch

Table 2: Estimation Results with All Three Adjustments, Basic Non-Nested Tree

ITEMS parameters	ORIGINAL RESULTS		ADJUSTMENT RESULTS		WEIGHT RESULTS		FREQUENCY RESULTS		ADJUSTMENT FREQUENCY RESULTS		ADJUSTMENT WEIGHT RESULTS		ADJUSTMENT FREQUENCY RESULTS		ALLFACTORS	
	estimate	abs	estimate	abs	estimate	abs	freq	freq	estimate	abs	estimate	abs	freq	freq	estimate	abs
Bus (B)																
ScheMis	-0.020	14.8	-0.020	14.8	-0.020	14.1	-0.020	14.8	-0.020	14.8	-0.020	14.1	-0.020	14.1	-0.020	14.1
WalkOrig	-0.112	16.6	-0.110	16.0	-0.110	16.0	-0.112	16.6	-0.110	16.0	-0.110	16.6	-0.110	16.0	-0.110	16.0
WalkDest	-0.050	11.8	-0.050	11.5	-0.050	11.0	-0.050	11.8	-0.050	11.5	-0.050	11.0	-0.050	11.0	-0.050	11.0
Fare	-0.326	8.9	-0.324	8.2	-0.327	5.6	-0.326	8.9	-0.324	8.2	-0.324	2.7	-0.327	5.6	-0.324	2.7
hConstant	-0.138	18.8	-0.132	15.8	-0.134	21.9	-0.137	18.8	-0.132	15.8	-0.132	8.0	-0.134	21.9	-0.132	8.0
hConstant	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0
hConstant	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0
Taxi (T)																
TRide	-0.053	5.7	-0.052	5.7	-0.056	2.4	-0.053	5.7	-0.052	5.7	-0.059	12.2	-0.056	2.4	-0.059	12.2
TFare	-0.968	26.9	-0.965	17.8	-0.923	22.6	-0.968	26.9	-0.966	17.8	-0.928	14.1	-0.923	22.6	-0.928	14.1
TSchedMis	-1.200	9.2	-1.230	8.2	-1.400	11.3	-0.926	4.0	-0.927	3.6	-1.400	6.8	-0.774	6.3	-0.779	3.8
TSchedMis	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0
Ride with Friend																
FRideExtra	-0.084	21.1	-0.084	21.0	-0.070	20.7	-0.084	21.1	-0.084	21.0	-0.066	7.5	-0.070	20.7	-0.066	7.5
FRide	-0.020	9.1	-0.019	7.3	-0.023	6.4	-0.020	9.1	-0.019	7.3	-0.022	2.8	-0.023	6.4	-0.022	2.8
FRideMisF	-0.012	2.5	-0.012	18.4	-0.028	24.8	-0.012	2.5	-0.012	18.4	-0.028	24.8	-0.012	2.5	-0.028	24.8
FRideMis	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0
FRideMis	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0
Ride with Friend NOT																
DRide	-0.072	13.3	-0.072	15.0	-0.077	13.5	-0.072	13.3	-0.072	15.0	-0.071	5.5	-0.077	13.5	-0.071	5.5
DRide	-1.440	13.9	-1.438	14.2	-1.460	12.4	-0.720	4.8	-1.467	4.9	-1.406	6.8	-0.388	3.4	-0.388	3.4
DRide	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0
Ride with Relation																
FRideExtra	-0.048	19.6	-0.048	16.4	-0.044	24.5	-0.048	19.6	-0.048	16.4	-0.044	8.0	-0.044	24.5	-0.044	8.0
FRide	-0.018	6.7	-0.017	9.0	-0.009	4.0	-0.018	6.7	-0.017	9.0	-0.009	1.8	-0.009	4.0	-0.009	1.8
FRideMisF	-0.025	21.0	-0.025	19.0	-0.025	20.2	-0.025	21.0	-0.025	19.0	-0.048	9.1	-0.025	20.2	-0.048	9.1
FRideMis	-0.240	3.2	-0.240	2.8	-0.009	0.1	0.270	2.5	-0.240	2.8	0.009	0.0	-0.009	0.1	0.009	0.1
FRideMis	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0
Ride with Relation NOT																
HRide	-0.041	14.3	-0.041	9.4	-0.047	12.3	-0.041	14.3	-0.041	9.4	-0.043	4.4	-0.047	12.3	-0.043	4.4
HSchedMis	-1.300	9.2	-1.309	6.4	-1.440	9.6	-0.667	0.3	-1.332	0.2	-1.325	3.7	-0.667	0.3	-0.667	0.3
HSchedMis	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0	0.000	0.0
sample size																
total observations	737		737		737		737		737		737		737		737	
total observations	4212		4212		4212		4212		4212		4212		4212		4212	
goodness of fit																
Final likelihood	-2054.9		-2054.9		-2054.9		-2054.9		-2054.9		-2054.9		-2054.9		-2054.9	
Final likelihood	-2752.2		-2754.7		-2752.2		-2752.2		-2754.7		-2752.2		-2754.7		-2752.2	
Δ205	0.148		0.148		0.148		0.148		0.148		0.148		0.148		0.148	

Table 3: T-statistics for the Coefficients Using Original Data with no Adjustments, Basic Tree





